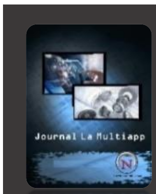
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Magneto Thermosolutal Convection in a Compressible Viscoelastic Fluid

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Abstract

In the presence of a magnetic field, a thermosolutal convection is postulated to occur in a compressible Rivlin-Ericksen viscoelastic fluid in a porous media. The dispersion relation is found by using the linear stability theory and the normal mode analysis approach, respectively. In the scenario of stationary convection, it was discovered that compressibility, magnetic fields, and steady solute gradients all serve to delay the beginning of the convection process, but medium permeability serves to speed up the beginning of the convection process. In addition to this, it has been discovered that the system is reliable for $\frac{1}{G-1}\frac{c_p\alpha\kappa}{\nu} \leq \frac{4\pi^2}{P_1}$ *and under the condition* $\frac{1}{G-1}\frac{c_p\alpha\kappa}{\nu} > \frac{4\pi^2}{P_1}$, *The system goes into an unstable state. Overstability has also been looked at from the perspective of a scenario in which sufficient circumstances are met to rule out the possibility of the phenomenon occurring. It has been discovered that the steady gradient of the solute and the magnetic field both induce oscillatory modes into the system.*

Introduction

According to distinct hydrodynamic and hydromagnetic assumptions, Chandrasekhar's classic monograph summarizes the theoretical and experimental findings on the initiation of thermal convection (Be'nard convection) in a fluid layer. In a fluid layer, same observations were observed (1981). Double-diffusive convection is an intriguing phenomenon with intriguing intricacies, and it has direct application to geophysics and astronomy. Solute gradients and free boundaries are preferable to rigid boundaries when studying double-diffusive convection because the circumstances under which convective motion is significant in double-diffusive convection are often far away from the examination of a single component fluid and rigid boundaries. A multitude of geophysical and astrophysical situations fall into this category, as do the lowest levels of Earth's atmosphere and astrophysics. To his credit, Veronis was the first to look at thermohaline convection in a fluid layer heated from below and exposed to a continuous salinity gradient (1965). The physics of the stellar situation are quite similar to those of salt because helium increases density and diffuses more slowly than heat. The ionosphere and the uppermost layer of the atmosphere are critical to atmospheric physics and astrophysics, thus finding a solution to this issue is critical. With regards to convection, oceanography, limnology, and engineering are all affected by it.

As the system becomes more compressible, the governing equations of the system become much more difficult to understand. A set of equations governing compressible fluid flow was simplified by Spiegel and Venonis (1960) on the basis of two key assumptions: (a) the depth of the fluid layer is much smaller than the scale height defined by them, and (b) the fluctuations

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in temperature, density, and pressure introduced by motion do not exceed their total static variations. In order to achieve this, they assumed that (a) the depth of the fluid layer is much less than the scale height (as defined by them), and (b) with the exception that the static temperature gradient is replaced by its excess over the adiabatic one, the flow equations for incompressible fluids under the aforementioned assumptions are the same as those for compressible fluids. When a magnetic field and rotation are present in compressible fluids, thermal instability may arise. Sharma (1977) studied this phenomenon. These findings were based on an assessment of this magnitude.

You may categorize a wide range of common and rare substances, including paints, polymers, and plastics, as viscoelastic fluids, including the earth's lithosphere, silicic magma, and saturated soil. Many researchers have been working on figuring out how viscoelastic materials move. Viscoelastic materials are increasingly being used in a variety of manufacturing and processing industries, as well as in geophysical fluid dynamics, chemical technologies, and the petroleum industry. A theoretical model for fluids with viscoelastic features was presented by Oldroyd (1958). Research in viscoelastic fluid dynamics has hitherto focused on waves and stability. That's because polymers and electrochemistry both employ viscoelastic fluids heavily. It is possible for viscoelastic fluids to exhibit distinct effects due to instability and other causes, unlike Newtonian fluids. According to Toms and Strawbridge's (1953) experiment, the Oldroyd fluid's theoretical model may be accurately reproduced by diluting methyl methacrylate in butyl acetate. The experiment's results led to this conclusion. When two Oldroydian viscoelastic fluids of the same density are stacked on top of one another, Sharma and Sharma in 1978 looked at the stability of the planar interface. Many elastico-viscous fluids cannot be described by Oldroyd's constitutive relations. An elastico-viscous fluid, such as the Rivlin-Ericksen (1955) fluid, is one example of this kind. The Rivlin-Ericksen fluid (1955) may be explained using the constitutive equations, which were first described in 1955.

$$\begin{aligned} S = -pI + \mu A_1 + \mu' A_2 + \mu^{ii} A_1^2 + \mu^{iii} A_2^2 + \mu^{iv}(A_1 A_2 + A_2 A_1) + \mu^{v}(A_1^2 A_2 + A_2 A_1^2) \\ + \mu^{vi}(A_1 A_2^2 + A_2^2 + A_1) + \mu^{vii}(A_1^2 A_2^2 + A_2^2 A_1^2) \end{aligned} \tag{1}$$

where S is the Cauchy stress tensor, ‘p’ is an arbitrary hydrostatic pressure, I is the unit tensor and μ′s are polynomial functions of the traces of the various tensors occurring in the representation, matrices ‘A_1’ and ‘A_2’ are defined by

$$[A_1]_{ij} = (q_{i,j} + q_{j,i}) \tag{2}$$

and

$$[A_2]_{ij} = \frac{\partial [A_1]_{ij}}{\partial t} + q_p [A_1]_{ij,p} + [A_1]_{ip} q_{p,j} + [A_1]_{pj} q_{p,i} \tag{3}$$

‘q_p’ being velocity vector.

On neglecting the squares and products of ‘A_2’, we have

$$S = -pI + \mu A_1 + \mu' A_2 + \mu^{ii} A_1^2 \quad , \tag{4}$$

where μ, μ^{i} *and* μ^{ii} are three material constants. It is customary to call μ , the coefficient of ordinary viscosity, μ' the coefficient of viscoelasticity *and* μ^{ii}, the coefficient of cross-viscosity. The μ, μ^{i} *and* μ^{ii} are general functions of temperature and material properties. For many fluids such as aqueous solution of polycrylamid and poly-isobutylene, μ, μ^{i} *and* μ^{ii} may be taken as constants. Polymers such as these and others are used in the manufacturing of components for aircraft and spacecraft, as well as tires, belt conveyers, ropes, cushions, seats, foams, plastics, technical equipment, adhesives, and contact lenses. Polymers are also employed in the production of contact lenses. Polymers have recently found use in a variety of sectors, including agriculture, communication devices, and biological research, among others. The hydromagnetic stability of two Rivlin-Ericksen elastico-viscous superposed conducting fluids was investigated by Sharma and Kumar (1997). In their study, Kumar and Singh (2006) investigated how the presence of suspended particles affects the stability of two superposed Rivlin-Ericksen viscoelastic fluids. They did this by conducting an evaluation of the effect of suspended particles on the stability of the system. In a separate work, Kumar et al. (2007) investigated the hydrodynamic and hydomagnetic stability of two stratified Rivlin-Ericksen elastic-viscous superposed fluids. These fluids were superposed one on top of the other. After each of the earlier studies, the researchers had the impression that the medium did not contain any holes.

In recent years, researchers have focused a great deal of attention on investigating how fluids move through porous media, notably geophysical fluid dynamicists. In the field of geophysics, one example of this may be seen in the process of extracting crude oil from the pores of reservoir rocks. One could find a significant number of geophysics-related applications in a book written by Phillips. [Citation needed] (1991). In accordance with Darcy's law, one may calculate the overall effect that a fluid has slowly seeping through the pores of rock has. As a direct consequence of this, the conventional viscous component that is found in equations that describe the motion of fluids is changed to the resistance term $\left[-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)\vec{q}\right]$, for Rivlin-Ericksen elastico-viscous fluid, where μ and μ′ are the viscosity and viscoelasticity of the fluid, k_1 is the medium permeability and $\vec{q}$ is the Darcian (filter) velocity of the fluid. People often believe that comets are made up of a dusty "snowball" that is composed of a combination of frozen gases. Comets go through a transition from their solid state to their gaseous state and back again as they travel through space. According to McDonnel (1978), the characteristics of comets, meteorites, and interplanetary dust suggest that porosity plays an important role in the context of study on astrophysics. The question of whether or not superposed Rivlin-Ericksen elastic-viscous fluids that are pierced with suspended particles may remain stable in a porous media was investigated by Kumar (2000). Researchers Kumar and colleagues (2004) investigated the instability caused by the superposition of two rotating viscoelastic (Rivlin-Ericksen) fluids with suspended particles in a porous media. They came to the conclusion that the particles were to blame for the instability. In a different piece of research, Kumar et al. (2005) investigated the MHD instability of rotating superposed Rivlin-Ericksen viscoelastic fluids that were traveling over porous surfaces. In the research that Aggarwal & Dixit (2017), they investigated the thermosolutal instability of a layer of elastico-viscous fluid in a porous media that included suspended particles. This instability was caused by the effect of compressibility.

The purpose of this research is to illustrate the effect that a magnetic field has on the thermosolutal convection of a compressible Rivlin-Ericksen elastico-viscous fluid as it moves through a porous medium. This is our primary objective, and we have selected it bearing in mind the significance of flow through porous media in the fields of geophysics, soil sciences, ground-water hydrology, atmospheric physics, astrophysics, and the applications stated before. The investigation starts with Section 2, which formulates the issue for the Rivlin-Ericksen viscoelastic electrically conducting compressible fluid using the Boussinesq approximation, the Spiegel and Veronis (1960) assumptions, the linearized theory, and the perturbation theory. This section is where the investigation gets underway. In Section 3, a dispersion relation is constructed by making use of the normal mode methodology. Within the context of stationary convection, Section 4 presents an analytical investigation of the impacts of compressibility, magnetic field, constant solute gradient, and medium permeability. In the fifth and last part of this series, many important theorems pertaining to the system's stability are studied, along with additional requirements for the absence of overstability.

Formulation of the Problem and Perturbation Equations

In this example, we take into account an infinite, horizontal, electrically conducting, compressible Rivlin-Ericksen elastico-viscous fluid layer of thickness d in a porous medium. The fluid layer is heated and soluted from below in such a way that the temperatures, densities, and solute concentrations at the bottom surface remain constant $z = 0$ are $T_0, \rho_0,$ and C_0, and at the upper surface $z = d$ are T_d , ρ_d, and C_d, respectively, with the z −axis being taken as vertical, and that a uniform adverse temperature gradient ($\beta = |dT/dz|$) and a uniform solute gradient ($\beta' = |dC/dz|$) are maintained. This layer is acted on by a gravity field $\vec{g}(0,\ 0,\ -g)$ and uniform vertical magnetic field $\vec{H}(0,\ 0,\ H)$.

Spiegel and Veronis (1960) defined f as any one of the state variables (pressure p, density ρ or temperature$T$) and expressed these in the form

$$f(x,\ y,\ z,\ t) = f_m + f_0(z) + f'(x,\ y,\ z,\ t) \tag{5}$$

where f_m is the constant space average of f, f_0 is the variation in the absence of motion and f' is the fluctuation resulting from motion.

The initial state is, therefore, a state in which the density, pressure, temperature, solute concentration and velocity at any point in the fluid are given by

$\rho = \rho(z),\ p = p(z),\ T = T(z),\ C = C(z),\ \vec{q} = 0$ respectively,

where $T(z) = T_0 - \beta z,\ C = C_0 - \beta' z,\ p(z) = p_m - g\int_0^z (\rho_m + \rho_0)dz,$

$$\rho(z) = \rho_m[1 - \alpha_m(T - T_m) + \alpha'_m(C - C_m) + K_m(p - p_m)],$$

$$\alpha_m = -\left(\frac{1}{\rho}\frac{\partial \rho}{\partial T}\right)_m (= \alpha, say), \alpha'_m = -\left(\frac{1}{\rho}\frac{\partial \rho}{\partial C}\right)_m (= \alpha', say),$$

$$K_m = \left(\frac{1}{\rho}\frac{\partial \rho}{\partial p}\right)_m. \tag{6}$$

Let $\delta\rho, \delta p, \theta, \gamma, \vec{q}(u, v, w)\ and\ \vec{h}(h_x, h_y, h_z)$denote, respectively, the perturbations in density ρ, pressure p, temperature T, solute concentration C, velocity $\vec{q}(0, 0, 0)$ and magnetic field $\vec{H}(0,0,H)$. Then the linearized hydromagnetic perturbation equations relevant to the problem are

$$\frac{1}{\varepsilon}\frac{\partial \vec{q}}{\partial t} = -\frac{1}{\rho_m}\nabla\delta p + \vec{g}\frac{\delta\rho}{\rho_m} - \frac{1}{k_1}\left(\nu + \nu'\frac{\partial}{\partial t}\right)\vec{q} + \frac{\mu_e}{4\pi\rho_m}(\nabla \times \vec{h}) \times \vec{H}, \tag{7}$$

$$\nabla.\vec{q} = 0, \tag{8}$$

$$\varepsilon\frac{\partial \vec{h}}{\partial t} = (\vec{H}.\nabla)\vec{q} + \varepsilon\eta\nabla^2\vec{h}\,, \tag{9}$$

$$\nabla.\vec{h} = 0\,, \tag{10}$$

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$$E\frac{\partial\theta}{\partial t} = \left(\beta - \frac{g}{c_p}\right)w + \kappa\nabla^2\theta\,, \qquad (11)$$

$$E'\frac{\partial\gamma}{\partial t} = \beta' w + \kappa'\nabla^2\gamma\,. \qquad (12)$$

Here $\nu, \nu', c_p, \kappa\ and\ \kappa'$ stand for kinematic viscosity, kinematic viscoelasticity, specific heat at constant pressure, thermal diffusivity and solute diffusivity respectively.

$$E = \varepsilon + (1-\varepsilon)\frac{\rho_s c_s}{\rho_0 c}$$

where ρ_s, c_s and ρ_0, c are the densities and specific heats of the solid (porous matrix) and fluid respectively. $E'$is a constant analogous to E but corresponding to the solute rather to the heat.

The equation of state is

$$\rho = \rho_m[1 - \alpha(T - T_0) + \alpha'(C - C_0)]\,, \qquad (13)$$

where α is the coefficient of thermal expansion and α'analogous the solute coefficient. The suffix zero refers to the values at the reference level $z = 0$. The change in density $\delta\rho$, caused by the perturbations θ and γ in temperature and concentration, is given by

$$\delta\rho = -\rho_m(\alpha\theta - \alpha'\gamma). \qquad (14)$$

Writing the scalar components of Eq (7), eliminating u, v, h_x, h_y and δp between them by using Eqs (8) – (12), we obtain

$$\frac{1}{\varepsilon}\frac{\partial}{\partial t}\nabla^2 w = g\alpha\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) - g\alpha'\left(\frac{\partial^2\gamma}{\partial x^2} + \frac{\partial^2\gamma}{\partial y^2}\right) - \frac{1}{k_1}\left(\nu + \nu'\frac{\partial}{\partial t}\right)\nabla^2 w + \frac{\mu_e H}{4\pi\rho_m}\nabla^2\frac{\partial h_z}{\partial z}, (15)$$

$$\varepsilon\left(\frac{\partial}{\partial t} - \eta\nabla^2\right)h_z = H\frac{\partial w}{\partial z}\,, \qquad (16)$$

$$\left(E\frac{\partial}{\partial t} - \kappa\nabla^2\right)\theta = \left(\beta - \frac{g}{c_p}\right)w, \qquad (17)$$

$$\left(E'\frac{\partial}{\partial t} - \kappa'\nabla^2\right)\gamma = \beta' w\,. \qquad (18)$$

Consider the case in which both the boundaries are free and temperature, solute concentrations at the boundaries are kept constant. Then the boundary conditions appropriate to the problem are

$$w = \frac{\partial^2 w}{\partial z^2} = 0\,, \theta = 0\,, \gamma = 0\,, h_z = 0\ at\ z = 0\ and\ z = d. \qquad (19)$$

Dispersion Relation

Here we analyze the disturbances into normal modes and assume that the perturbation quantities are of the form

$$[w, \theta, h_z, \gamma] \\ = [W(z), \Theta(z), K(z), \Gamma(z)] e^{ik_x x + ik_y y + nt}, \qquad (20)$$

where k_x , k_y are wave numbers along $x -$ and $y -$directions respectively, $k\left(= \sqrt{k_x^2 + k_y^2}\right)$ is the resultant wave number and n is the growth rate which is, in general, a complex constant.

Using expression (20), Eqs (15) – (18) in non-dimensional form become

$$\frac{\sigma}{\varepsilon}(D^2 - a^2)W + g\frac{a^2 d^2}{\nu}(\alpha\Theta - \alpha'\Gamma) = -\frac{1}{P_1}(1 + F\sigma)(D^2 - a^2)W + \frac{\mu_e H d}{4\pi\rho_m \nu}(D^2 - a^2)DK,$$

(21)

$$(D^2 - a^2 - p_2\sigma)K = -\frac{Hd}{\varepsilon\eta}DW\,, \qquad (22)$$

$$(D^2 - a^2 - Ep_1\sigma)\Theta \\ = -\left(\frac{G-1}{G}\right)\frac{\beta d^2}{\kappa}W\,, \qquad (23)$$

$$(D^2 - a^2 - E'q\sigma)\Gamma \\ = -\frac{\beta' d^2}{\kappa'}W\,, \qquad (24)$$

where

$$a = kd, \sigma = \frac{nd^2}{\nu}, x = x^* d, y = y^* d\,, z = z^* d \; and \; D = \frac{d}{dz^*}\,, p_1 \\ = \frac{\nu}{\kappa} \; is \; the \; Prandtl \; number,$$

$p_2 = \frac{\nu}{\eta}$ is the magnetic Prandtl number and $q = \frac{\nu}{\kappa'}$ is the Schmidt number, $P_1 = \frac{k_1}{d^2}$ is the dimensionless medium permeability, $F = \frac{\nu'}{d^2}$ is the dimensionless kinematic viscoelasticity and $G = \frac{c_p\beta}{g}$ is the dimensionless compressibility parameter. We shall suppress the star $(*)$ for convenience hereafter.

Eliminating $\Theta, K \; and \; \Gamma$ between Eqs (21) – (24), we obtain

$$(D^2 - a^2)(D^2 - a^2 - Ep_1\sigma)(D^2 - a^2 - E'q\sigma)\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_1}(1 + F\sigma)\right](D^2 - a^2 - p_2\sigma)W \\ = Ra^2\left(\frac{G-1}{G}\right)(D^2 - a^2 - E'q\sigma)(D^2 - a^2 - p_2\sigma)W \\ - Sa^2(D^2 - a^2 - Ep_1\sigma)(D^2 - a^2 - p_2\sigma)W \\ - \frac{QD^2}{\varepsilon}(D^2 - a^2)(D^2 - a^2 - Ep_1\sigma)(D^2 - a^2 - E'q\sigma)W\,, \qquad (25)$$

where $R = \frac{g\alpha\beta d^4}{\nu\kappa}$ is the Rayleigh number, $S = \frac{g\alpha'\beta' d^4}{\nu\kappa'}$ is the solute Rayleigh number and $Q = \frac{\mu_e H^2 d^2}{4\pi\rho_0 \nu\eta}$ is the Chandrasekhar number.

Using expression (20), the boundary conditions (19), in non-dimensional form, transform to

$$W = D^2W = 0\,,\Theta = 0\,,\Gamma = 0\,,K = 0\ at\ z = 0\ and\ z = 1. \qquad (26)$$

Using the boundary conditions (26), it can be shown with the help of Eqs (21) – (24) that all the even order derivatives of W must vanish at $z = 0$ and $z = 1$. Hence the proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z\,, \qquad (27)$$

where W_0 is a constant. Substituting the proper solution (27) in Eq (25), we obtain the dispersion relation

$$R_1 = \frac{\frac{G}{G-1}\left[\begin{array}{c}(1+x)(1+x+iEp_1\sigma_1)(1+x+iE'q\sigma_1)(1+x+ip_2\sigma_1)\left\{\frac{i\sigma_1}{\varepsilon}+\frac{1}{P}(1+i\sigma_1\pi^2F)\right\}\\ +S_1x(1+x+iEp_1\sigma_1)(1+x+ip_2\sigma_1)+\frac{Q_1}{\varepsilon}(1+x)(1+x+iEp_1\sigma_1)(1+x+iE'q\sigma_1)\end{array}\right]}{x(1+x+iE'q\sigma_1)(1+x+ip_2\sigma_1)},$$

(28)

where

$$R_1 = \frac{R}{\pi^4}\,, S_1 = \frac{S}{\pi^4}\,, x = \frac{a^2}{\pi^2}\,, P = \pi^2 P_1\,, i\sigma_1 = \frac{\sigma}{\pi^2}\ and\ Q_1 = \frac{Q}{\pi^2}.$$

The Stationary Convection

When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma = 0$. Putting $\sigma = 0$, the dispersion relation (28) reduces to

$$R_1 = \frac{G}{G-1}\left[\frac{(1+x)^2}{xP} + S_1 + \frac{Q_1(1+x)}{x\varepsilon}\right], \qquad (29)$$

which expresses modified Rayleigh number R_1 as a function of dimensionless wave number x and the parameters $G\,, S_1\,, Q_1$ and P. For fixed $P\,, Q_1\,, S_1$ and G (accounting for compressibility effect) also be kept fixed.

Then we find that

$$\bar{R}_c = \left(\frac{G}{G-1}\right) R_c\,, \qquad (30)$$

where $\bar{R}_c$ and R_c denote respectively the critical Rayleigh numbers in the presence and absence of compressibility. $G > 1$ is relevant here. In the presence of compressibility, the instances $G = 1$ and $G < 1$ correspond, respectively, to negative and infinite values of the critical Rayleigh numbers. Since they are not relevant to the current topic, we will not consider them. Because of this, the impact of compressibility is to delay the beginning of the thermosolutal convection process. Therefore, compressibility contributes to the maintenance of stability.

Equation (29) makes it abundantly clear that when it comes to stationary convection, a Rivlin-Ericksen elastico-viscous fluid operates in the same manner as a typical Newtonian fluid. The result of Equation (29) is

$$\frac{dR_1}{dP} = -\left(\frac{G}{G-1}\right)\frac{(1+x)^2}{xP^2}\,, \tag{31}$$

$$\frac{dR_1}{dS_1} = \frac{G}{G-1}\,, \tag{32}$$

and

$$\frac{dR_1}{dQ_1} = \left(\frac{G}{G-1}\right)\frac{(1+x)}{x\varepsilon}\,, \tag{33}$$

With Rivlin-elastico-viscous Ericksen's fluid for stationary convection, this reveals that the medium permeability hastens the commencement of convection, while steady solute gradient and magnetic field postpone the onset of convection.

The critical Rayleigh number is derived from Eq (29) by putting $\frac{dR_1}{dx} = 0$, i.e.

$$\left(\frac{G}{G-1}\right)\left[\frac{x^2-1}{x^2P} - \frac{Q_1}{\varepsilon x^2}\right] = 0$$

or

$$x^2 = 1 + \frac{PQ_1}{\varepsilon}\,. \tag{34}$$

with $x = \sqrt{1+\frac{PQ_1}{\varepsilon}}$, as wave number is always positive, Eq (29) will yield the required critical Rayleigh number.

Some Important Theorems

Theorem 1: The system is stable for $G < 1$.

Proof: Multiplying Eq (21) by W^*, the complex conjugate of W, integrating over the range of z and making use of Eqs (22) – (24) together with boundary conditions (26), we obtain

$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_1}(1+F\sigma)\right]I_1 - \left(\frac{1}{G-1}\right)\frac{c_P\alpha\kappa a^2}{\nu}(I_2 + Ep_1\sigma^* I_3) + \frac{g\alpha'\kappa' a^2}{\nu\beta'}(I_4 + E'q\sigma^* I_5) + \frac{\mu_e\varepsilon\eta}{4\pi\rho_m\nu}(I_6 + p_2\sigma^* I_7) = 0\,, \tag{35}$$

where

$$I_1 = \int_0^1 (|DW|^2 + a^2|W|^2)dz\,,\quad I_2 = \int_0^1 (|D\Theta|^2 + a^2|\Theta|^2)dz,\quad I_3 = \int_0^1 (|\Theta|^2)dz,$$

$$I_4 = \int_0^1 (|D\Gamma|^2 + a^2|\Gamma|^2)dz\,,\quad I_5 = \int_0^1 (|\Gamma|^2)dz\,,\quad I_6 = \int_0^1 (|D^2K|^2 + 2a^2|DK|^2 + a^4|K|^2)dz,$$

$$I_7 = \int_0^1 \left(|DK|^2 + a^2|K|^2\right)dz\,, \tag{36}$$

where σ^* is the complex conjugate of σ and the integrals $I_1 - I_7$ are all positive definite.

Putting $\sigma = \sigma_r + i\sigma_i$ and then equating real and imaginary parts of Eq (35), we get

$$\sigma_r\left[\left(\frac{1}{\varepsilon}+\frac{F}{P_1}\right)I_1 - \left(\frac{1}{G-1}\right)\frac{c_P\alpha\kappa a^2}{\nu}Ep_1I_3 + \frac{g\alpha'\kappa'a^2}{\nu\beta'}E'qI_5 + \frac{\mu_e\eta\varepsilon}{4\pi\rho_m\nu}p_2I_7\right] = -\left[\frac{1}{P_1}I_1 - \left(\frac{1}{G-1}\right)\frac{c_P\alpha\kappa a^2}{\nu}I_2 + \frac{g\alpha'\kappa'a^2}{\nu\beta'}I_4 + \frac{\mu_e\eta\varepsilon}{4\pi\rho_m\nu}I_6\right], \tag{37}$$

and

$$\sigma_i\left[\left(\frac{1}{\varepsilon}+\frac{F}{P_1}\right)I_1 + \left(\frac{1}{G-1}\right)\frac{c_P\alpha\kappa a^2}{\nu}Ep_1I_3 - \frac{g\alpha'\kappa'a^2}{\nu\beta'}E'qI_5 - \frac{\mu_e\eta\varepsilon}{4\pi\rho_m\nu}p_2I_7\right] = 0. \tag{38}$$

It is evident from Eq (37) that if $G < 1$, σ_r is negative meaning thereby the stability of the system.

Theorem 2: The modes may be oscillatory or non-oscillatory in contrast to the case of no magnetic field and in the absence of stable solute gradient where modes are non-oscillatory, for $G > 1$.

Proof: Equation (38) yields that $\sigma_i = 0$ or $\sigma_i \neq 0$, which means that modes may be non-oscillatory or oscillatory. In the absence of stable solute gradient and magnetic field, Eq (38) gives

$$\sigma_i\left[\left(\frac{1}{\varepsilon}+\frac{F}{P_1}\right)I_1 + \left(\frac{1}{G-1}\right)\frac{c_P\alpha\kappa a^2}{\nu}Ep_1I_3\right] = 0, \tag{39}$$

and the terms in brackets are positive definite when $G > 1$. Thus $\sigma_i = 0$, hence, oscillatory modes are not permitted, and the idea of exchange of stabilities was successfully implemented. Both the steady gradient of solute concentration and the presence of the magnetic field are responsible for the introduction of oscillatory modes into the system; these modes were not present when neither of these factors was present.

Theorem 3: The system is stable for $\frac{1}{G-1}\frac{c_p\alpha\kappa}{\nu} \leq \frac{4\pi^2}{P_1}$and the condition $\frac{1}{G-1}\frac{c_p\alpha\kappa}{\nu} > \frac{4\pi^2}{P_1}$, the system becomes unstable.

Proof: From Eq (38) it is clear that σ_i is zero when the quantity multiplying it is not zero and arbitrary when this quantity is zero.

If $\sigma_i \neq 0$, Eq (37) upon utilizing (38) and the Rayleigh-Ritz inequality gives

$$\left[\frac{4\pi^2}{P_1} - \frac{1}{G-1}\frac{c_p\alpha\kappa}{\nu}\right]\int_0^1 |W|^2dz + \frac{\pi^2+a^2}{a^2}\left\{\frac{g\alpha'\kappa'a^2}{\nu\beta'}I_4 + \frac{\mu_e\varepsilon\eta}{4\pi\rho_m\nu}I_6 + 2\sigma_r\left(\frac{1}{\varepsilon}+\frac{F}{P_1}\right)\right\} \leq 0, \tag{40}$$

since the minimum value of $\frac{\left(\pi^2+a^2\right)^2}{a^2}$ with respect to a^2 is $4\pi^2$.

Now, let $\sigma_r \geq 0$, we necessarily have from inequality (40) that

$$\frac{1}{G-1}\frac{c_p\alpha\kappa}{\nu} > \frac{4\pi^2}{P_1}. \tag{41}$$

Hence, if

$$\frac{1}{G-1}\frac{c_p\alpha\kappa}{\nu} \leq \frac{4\pi^2}{P_1}, \tag{42}$$

then $\sigma_r < 0$. Therefore, the system is stable.
Therefore, under condition (42), the system is stable and under condition (41) the system becomes unstable.

Theorem 4: $Ep_1 > p_2 \; and \; Ep_1 > E'q$, are the sufficient conditions for the non-existence of overstability.

Proof: For overstability, we wish to determine the critical Rayleigh number for the onset of instability via a state of pure oscillations, it is suffice to find conditions for which equation (28) will admit of solutions with σ_1 real.

Equating real and imaginary of Eq (28) and eliminating R_1between them, we obtain

$$A_2c_1^2 + A_1c_1 + A_0 = 0\,, \tag{43}$$

where we have put $1 + x = b\,, c_1 = \sigma_1^2$,

and

$$A_2 = \left[\frac{EE'p_1p_2^2q^2b}{P} + b^2p_2^2E'^2q^2\left(\frac{1}{\varepsilon} + \frac{\pi^2F}{P}\right)\right],$$

$$A_1 = \left[\frac{p_1q^2EE'b^3}{P} + \frac{Ep_1p_2^2b^3}{P} + b^4\left(q^2E'^2 + p_2^2\right)\left(\frac{1}{\varepsilon} + \frac{\pi^2F}{P}\right) + S_1(b-1)bp_2^2(Ep_1 - E'q)\frac{Q_1}{\varepsilon}b^2q^2E'^2(Ep_1 - p_2)\right],$$

$$A_0 = b^6\left(\frac{1}{\varepsilon} + \frac{\pi^2F}{P}\right) + \frac{b^5Ep_1}{P} + S_1(b-1)b^3(Ep_1 - E'q) + \frac{Q_1}{\varepsilon}b^4(Ep_1 - p_2).$$

Since σ_1 is real for overstability, both the values of $c_1 (= \sigma_1^2)$ are positive. Equation (43) is quadratic in c_1 and does not involve any of its roots to be positive, if

$$Ep_1 > p_2 \; and \; Ep_1 > E'q\,. \tag{44}$$

Thus $\boldsymbol{Ep_1 > p_2 \; and \; Ep_1 > E'q}$ **are the sufficient conditions for the non-existence of over stability, the violation of which does not necessarily imply the occurrence of over stability**.

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